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## BINARY AND TERNARY STRUCTURES OF THE EVOLUTIONS IN THE UNIVERSE $(2 \times 3 \times 2 \times \ldots$-WORLD) IV THE ENTROPY DESCRIPTION OF EVOLUTION

## Summary

This is the fourth part of the papers which is written under the same title [30, 31, 16]. In the first and second parts, we have seen that binary and ternary structures can describe evolutions of systems, for example, quarks, atoms, galaxies, RNA, DNA and languages. In the third paper, we have given the evolution of languages and shown that it has an intimate connection to that in physics. In this part we shall develop a "general evolution theory" for the systems with binary and ternary structures at first. Then we will show how evolutionary systems create so called complexity systems as the border of the evolutionary system. We consider the evolution based on the following principle:

## The principle of evolution

(1) Every system in this universe must obey the law of increase of entropy (Boltzmann's principle) ([35]).
(2) Evolutionary systems perform against the Boltzmann principle (Schrödinger's principle or Bergson's philosophy) ([3])

Keywords and phrases: entropy, random walk, the evolution of the universe, non-commutative Galois theory

## Contents

Introduction

1. The entropy of evolution
2. The several evaluation processes (Entropy transformation)
3. Simple random walks in evolutions (Seeds)
4. Self avoiding random walks in evolutions (Self organization: The power law distribution)
5. The fractal structure of evolutionary system
6. Evolutions in physics/cosmology
7. Evolutions in polymer physics (de Gennes theory on polymers)
8. Evolutions in biology

Summary and problems (A method of system analysis to physics/cosmology)

## Introduction

We have found the structure of non-commutative binary and ternary Galois extensions in evolutionary systems and introduced a concept of the BTBB-structure in various fields in a unified manner ([30], [31]). Here we will introduce a concept of evolution entropy and describe the theory of evolutions in terms of the entropy. Then we can find the origins of fractal/chaotic structures and the power law distributions.

## The total evolutions to be considered

We begin with recalling the evolution of the universe. We state the BTBB-structure of the evolutions in the universe in Figure 1.

We give a short description on the evolutions and their BTBB-structure:
(I) From Big Bang time to $10^{-33}$ second: Quarks are born and baryons, mesons are created (These physics have the BTBB-structure (Section 3 in Part I [30])).
(II) From $10^{-33}$ second to 3 -minutes: The atoms C, O, N, $\ldots$, Fe are created from H and He. The typical BTBB-structure can be observed.
(III) From 3-minutes to 5 billion years: In this long term, stars and galaxies are created. The BTBB-structure cannot be observed. We will give the reason for this in this part (Section 8).
(IV) After 10 billion years: The life is born and RNA, DNA and proteins are created. The typical BTBB-structure can be observed.
$(\mathrm{V})$ The human beings are created and the language structure is created. The BTBBstructure and the complexity structure are discussed in Part III([16]).

The scheme of the evolution is given in Figure 2.

## The total hierarchy structure of evolutions

(I) The generations of the BTBB-system and the complexity system.
(II) The repetition of the process (I) creates the total hierarchy structure of the evolutionary system:
(1) We denote the set of seeds by $\mathscr{L}_{0}$
(2) We denote the system generated by the BTBB-structure and its complexity system by $\mathscr{L}_{1}$.


Fig. 1. (a) Evolutions of the universe (b) The total evolutionary tree.


Fig. 2. The scheme of the evolution.
(3) Replacing $\mathscr{L}_{0}$ with $\mathscr{L}_{1}$ and we follow the processes (1), (2). Then we obtain the system $\mathscr{L}_{2}$.
(4) Repeating this process we obtain the system $\mathscr{L}_{n}(n=1,2, \ldots)$ which we can call the system $\mathscr{L}_{n}$, the evolutionary system of level $n$.

Remark: We can observe the fact that the evolution of (I) is drastic and new and that the evolution in (II) is systematized based on the evolution in (I). We can introduce the "curriculum vitae" of the evolution. This will be discussed in the final section.

## 1. The entropy of evolution

Here we recall some basic facts on entropy and introduce the concept of "entropy of evolution([30]).

## The basic concepts of entropy

The concept of entropy is introduced by Boltzmann which describes the tendency of physical phenomena ([35]). The set of possible physical states is denoted by $X$. We put $S(X)=k \log \#(X)$, where $\#(X)$ is the number of states and $k$ is a positive constant. $S(X)$ is called the entropy of the states. We can describe the entropy in terms of probability $p_{j}\left(\sum_{j=1}^{M} p_{j}=1, p_{j}>0\right)$ :

$$
\begin{equation*}
S=-k \sum_{j=1}^{M} p_{i} \log p_{i} \tag{1}
\end{equation*}
$$

Next we state some basic properties:
(1) Addition formula: Let $\left\{p_{i}(A)\right\},\left\{p_{j}(B)\right\}$ be two probabilities. Put $p_{i j}(A B)=$ $p_{i}(A) p_{j}(B)$. Then we have a new probability $\left\{p_{i j}(A B)\right\}$. Then we have

$$
S_{A B}=S_{A}+S_{B}
$$

(2) The maximal distribution: For an entropy function $S$, putting $p_{j}=\exp \left(-b E_{i}\right) / Z B G$, where $Z B G=\sum_{i=1}^{M} \exp (b E j)$, we can obtain the distribution which realizes the maximum entropy ([34]).

Remark The entropy of the states of a simple random walk attains the maximal entropy.
Shannon entropy The Shannon entropy is defined by the events of information. We take a system of $N$ kinds of events of information: Namely when we are given $p_{1}, p_{2}, \ldots, p_{N}$ with $\sum_{i=1}^{M} p_{i}=1$, then the Shannon entropy is given in the same manner as (1).

## The entropy of evolution

Next we introduce a concept of the entropy of the evolution and consider the basic process of the evolution in the following manner:

Process (I): From the origin to the creation of "sea of seeds
Process (II): The evolution begins and the self organization starts.
We may draw the process in the following configuration:

$$
\left.* \underset{\mathrm{I}}{\Rightarrow} \begin{array}{|ccc}
\mathrm{x} & \mathrm{x} & \mathrm{x} \\
\mathrm{x} & \mathrm{x} & \mathrm{x} \\
\mathrm{x}
\end{array}\right] \underset{\mathrm{II}}{\Rightarrow} \begin{array}{|ccc|}
\hline 0000 \\
0000 & \mathrm{o} & \\
0 & \mathrm{o} & \mathrm{o}
\end{array}
$$

Figure 3

We notice that not necessary all seeds are chosen to create the organization. Such seeds are called symmetry breaking elements. We describe the entropy of the evolution in each process:

## Process I

By the big explosion, we have the following structure:
(1) The origin creates not necessarily unique seeds $\mid 0>_{a}$. Also $\mid 0>_{a}$ constitutes the states $\mid 0>_{a}=\left\{X_{j}^{(a)}\right\}$.
(2) We assume that $\left\{X_{j}^{(a)}\right\}$ behave completely randomly. Namely, $\left\{X_{j}^{(a)}\right\}$ make a simple random walk. Namely, it makes a Brownian motion.
(3) We introduce the entropy of the seeds $S(X)=k \log \#(X)$, where $X=\left\{X_{j}^{(a)}\right\}$. It is called the entropy of the evolution. We notice that $S(X)$ gives the "size" of evolution.

## Process II

We describe the evolution process. We begin with a time evolution from the big explosion. We take states $X=X(0)$ of seeds at the beginning. We consider the time evolution $X(t)(t>0)$, where time is the usual Newtonian time. The entropy at time $t$ is denoted by $S_{i n}(t)$. $S_{i n}(t)$ is called of evolution type, when

$$
S_{i n}(t) \geq S_{i n}\left(t^{\prime}\right) \quad(t<t)
$$



Figure 4
Here we assume the existence of the ambient space $X_{\text {out }}$ of the original space $X_{i n}$. We denote the entropy of $X_{i n}$ (resp. $X_{o u t}$ ) by $S_{i n}$ or $S_{o u t}$, respectively. Here we have to notice that the total entropy should increase by the Boltzmann principle. Hence there exists an additional entropy which is called "emission entropy" which is emitted from $S_{\text {in }}(t)$ to $S_{\text {out }}(t)$ and we have

$$
S_{\text {in }}(t)+S_{\text {out }}(t) \geq S(0)
$$

Next we give several examples:

## Example 1 (Big-Bang)

The universe begins with the Big Bang and its inflation. Here we have to notice that we have our visible world and its ambient world, invisible world (or the "dark world") at the same time. The states make a simple random walk (Process I). Then the structure of the universe is constructed. The original evolution entropy decreases and some amount of entropy should be emitted to the ambient space (Process II). The details will be given in Section 6.

## Example 2 (The universe)

We can discuss the evolution of the universe in an analogous manner. We may choose the Process I in various manners:
(1) The equilibrium state of protons and neutrons: We begin with the distribution of the proton p and neutron n . They make an equilibrium state between them: $p \Leftrightarrow n$. The entropy is given by the set of states of $p$ and $n$ in the early time of the Big Bang (Process I).
(2) The stars of the $\mathbf{3 . 8} \times 10^{5}$ years old universe. We may choose the distribution of stars of the $3.8 \times 10^{5}$ years old universe. They make a distribution of the blackbody radiation. Hence we see that they make a simple random walk.

Then the self organization starts and galaxies are created and their distribution makes that of a power law type. This will be discussed later (in Section 6) (Process II).

## Example 3 (Life thing)

Usually the life things with sex make the following mating process:


Figure 5
where $*$ is the birth of life. The entropy defined by the distribution of eggs and sperm (Process I). Here we have to notice the following fact: After mating only one (or very few) egg survives and remained eggs die. Then the egg-division happens and the body construction begins (Process II).

## Example 4 (Polymers)

The seeds of the evolution are the total set of monomers:


Figure 6
The entropy might be chosen as the total set of monomers (Process I). We have the emission of the entropy/heat from the polymers to the ambient space (Process II).

## Example 5 (Natural language) ([16])

We have discussed the evolution of natural languages in Part III. There we may choose the origin as the intelligence of language creation. We may find seeds which are composed of sequences of alphabets (Process I). Here we choose the Shannon entropy. The process II creates sentences. We notice that the Shannon entropy gives the grammar of the natural language. The entropy of the information of words decreases in the process of the creation of sentences. When the constructed sentence is correct, the entropy becomes its minimum.

$$
* \begin{gathered}
\Rightarrow \\
\mathrm{I}
\end{gathered} \begin{gathered}
\mathrm{H} \mathrm{x}, \mathrm{y}, \mathrm{~d} \\
\mathrm{a} \mathrm{j} \mathrm{lo} \mathrm{jets} \\
\mathrm{Q}, \mathrm{u}, \mathrm{l}, \mathrm{r}
\end{gathered} \quad \Rightarrow \mathrm{II} \quad \begin{gathered}
\text { I read a book I study English } \\
\text { If he comes, I will start } \\
\text { She lives in Tokyo }
\end{gathered}
$$

Figure 7
Example 6 (The original Galois theory) ([37])
We will treat the evolution theory of the classical Galois theory in Part V.
(1) The origin is the choice of the field of mathematics: "Galois theory. (2) We choose the total set of the complex numbers (Process I), (3) The successive Galois extensions of the fields from the solutions $\left\{a_{j}:(j=1,2, \ldots, N)\right\}$ of an algebraic equation $f(x)=0: f\left(a_{j}\right)=0$, where the coefficient field is the rational numbers $\mathbb{Q}$ (Process II). The details will be given in Part V ([17]).

## 2. The several evolution processes (Entropy transformation)

In this section we introduce a concept of entropy transformation and discuss evolutions of general type.

## The general evolutionary System

We have restricted ourselves to the evolution which decreases the evolutionary entropy. Here we treat the evolutions of general type. We can classify the evolutions into the following types: (1) Top down type evolution (2) Bottom up type evolution (3) Mixed type evolution.

## (1) The top-down evolution

We have discussed this type evolution which can be observed in the self organization of systems. The evolution begins and the evolution entropy is prepared. Then the structures are constructed by decreasing its entropy.

$$
\left.* \underset{\mathrm{I}}{\Rightarrow} \begin{array}{|cccc}
\mathrm{x} & \mathrm{x} & \mathrm{x} \\
\mathrm{x} & \mathrm{x} & \mathrm{x} & \mathrm{x}
\end{array} \underset{\mathrm{II}}{\Rightarrow}\right]
$$

Figure 8
and finishes the evolution when it attains its minimum. Here we have to notice that the entropy creates "in a moment from the origin. Hence we can obtain the evolution entropy in the very beginning.

## (2) The bottom up evolution

This evolution creates the structure "slowly" and the entropy increases moderately. Finally the evolution attains the stable state at last. Then we may expect the following types of the end of the evolution: (i) Blow up of system, or (ii) Stable system.


Figure 9

## (3) The mixed type evolution

In order to treat the general evolutions including Darwinian evolutions, for example, we have to prepare the both types of evolutions at the same time. Typical examples can be observed in the stability of ecological system (for example Voltera-Lotka equation ([26])) (Also see Figure 12).

## Generating function of evolutionary system and the evolution transformation

We introduce a concept of generating function for the distribution of an evolutionary system and discuss the evolution in terms of generating functions. We choose an arbitrary evolutionary system at first. We take the distribution of the elements $g_{n}$ of $n$-th generation and consider the following formal power series:

$$
F(z)=1+g_{1} z+g_{2} z^{2}+g_{3} z^{3}+\ldots
$$

which is called the generating function of the evolutionary system. Correspondingly, we put

$$
F_{S}(z)=1+S\left(g_{1}\right) z+S\left(g_{2}\right) z^{2}+S\left(g_{3}\right) z^{3}+\ldots
$$

where $S\left(g_{n}\right)$ denotes the entropy of the state $g_{n}$.
Example 1. We take a system of a simple random walk. We describe the process in terms of the piecewise lines with a direction whose distribution of length $n$ is denoted by $g_{n}$ :
$g_{0}=\{$ the origin $\}$ (the path with length 0 )


Figure 10

Example 2. (1) The set of n-points.

$$
\begin{aligned}
& g_{0}=\text { the empty set } \\
& g_{1}=\bullet, g_{2}=\bullet \bullet, g_{3}=\bullet \bullet \bullet, \ldots
\end{aligned}
$$

(2) The set of binary (resp. ternary) elements:

$$
\begin{aligned}
& g_{0}=\emptyset, g_{1}=\square, g_{2}=\square, \ldots \\
& g_{0}=\emptyset, g_{1}=\square, g_{2}=\square
\end{aligned}
$$

## Entropy transformation of top-down type

We can introduce a concept of evolutionary transformation which transforms from one distribution, for example:

$$
\begin{aligned}
& F(z)=1+g_{1} z+g_{2} z^{2}+\ldots \\
& g_{1}=\bullet, g_{2}=\bullet \bullet, g_{3}=\bullet \bullet \bullet, \ldots
\end{aligned}
$$

to another distribution, for example:

$$
\begin{array}{r}
\hat{F}(z)=1+\hat{g}_{1} z+\hat{g}_{2} z^{2}+\hat{g}_{3} z^{3}+\ldots \\
\hat{g}_{1}=\bullet, \quad \hat{g}_{2}=\bullet, \quad \hat{g}_{3}=\boldsymbol{\bullet}, \ldots
\end{array}
$$

which is defined by $F: F(z) \rightarrow \hat{F}(z), F\left(g_{n}\right)=\hat{g}_{n}$ is called entropy transformation when $S\left(g_{n}\right) \geq S\left(\hat{g}_{n}\right)$. We give several examples.

Example 1 (Random walks) (1) (Sections 4, 5)
(1) For a simple random walk, we consider the distribution of possible $n$ - elements, which is denoted by $g_{n}$. Then we have the generating function:

$$
F(z)=1+g_{1} z+g_{2} z^{2}+g_{3} z^{3}+\ldots
$$

We see that $S\left(g_{n}\right)=(2 d)^{n}(d=$ the lattice dimension $)$.
(2) For a self avoiding random walk, we have the corresponding generating function:

$$
F(z)=1+\hat{g}_{1} z+\hat{g}_{2} z^{2}+\ldots
$$

We see that $\hat{g}_{n}=n^{\gamma}$, where $\gamma$ is the critical exponent. Then $F: F(z) \rightarrow \hat{F}(z)$ describes the self-organization.

## Example 2 (Borel transformation)

We consider a set of $n$ points: $\{1,2, \ldots, n\}$. We consider the permutations of $n$-points which are denoted by

$$
g_{n}=\left\{s(1), s(2), \ldots, s(n): s \in S_{n}\right\} .
$$

Then we see that $S\left(g_{n}\right)=n$ ! We make a congruence of these points into a single set which is denoted by $\hat{g}_{n}$. Then we see that $S\left(\hat{g}_{n}\right)=1$. Then the transformation can be understood as follows: The entropy transformation: $T: F(z) \rightarrow \hat{F}(z)$ gives $F\left(\sum n!z^{n}\right)=\sum \hat{z}^{n} . T$ is called Borel transformation.

## The evolution transformation of bottom-up type

In order to describe the egg division, or phase change from ice to water, we have to treat the inverse evolutionary transformation. $F: F(z) \rightarrow \hat{F}(z), F\left(g_{n}\right)=\hat{g}_{n}$ is called inverse entropy transformation when $S\left(g_{n}\right) \leq S\left(\hat{g}_{n}\right)$.

Example 1: The typical example of the inverse transformation can be observed in the cell division of life thing: This process happens when some energy is brought from the ambient space.

Example 2: Another example can be found in the self-organization of the phase transition from ice to water, for example.

## The transformation of mixed type

In order to treat the evolution process totally, we consider the transformations on the original space $X_{\text {in }}$ and the ambient space $X_{\text {out }}$ at the same time. For an evolution transformation: $F_{\text {in }}: F_{\text {in }}(z) \rightarrow F_{\text {in }}(\hat{z})$, we can associate the transformation: $F_{\text {out }}$ : $F_{\text {out }}(z) \rightarrow \hat{F}_{\text {out }}(z)$. We have to consider the both transform at the same time. In order to preserve the Boltzmann principle, we have the following conditions:

$$
\begin{aligned}
& S\left(g_{\text {in }}(n)\right)+S\left(g_{\text {out }}(n)\right) \leq S\left(\hat{g}_{\text {in }}(n)\right)+S\left(\hat{g}_{\text {out }}(n)\right), \\
& S\left(g_{\text {in }}(n)\right) \geq S\left(\hat{g}_{\text {in }}(n)\right), \quad S\left(g_{\text {out }}(n)\right) \leq S\left(g_{\text {out }}(n)\right) .
\end{aligned}
$$



Figure 11
Including the creation of the galaxies, the original Darwinian evolution theory and environmental problems, we have to consider several systems, $X_{1}, X_{2}, \ldots, X_{n}$ in a common ambient space $X_{\text {out }}$ and describe the interchange entropies between (1) $X_{i}$ and $X_{j},(2) X_{\text {in }}$ and $X_{\text {out }}$ at the same time ([6]).


Figure 12

## 3. Simple random walks in evolutions (Seeds)

In this section we recall some basic facts on simple random walk. As for simple random walk, see also ([7], [30]).
(Simple random walk on the $n$-dimensional lattice)
We take a simple random walk on the n-dimensional lattice.

simple random walk

$$
\mathbf{r}=\mathbf{a}_{1}+\mathbf{a}_{2}+\ldots+\mathbf{a}_{N}
$$

Figure 13
The random walk gives the distribution of Gaussian type:

$$
\begin{aligned}
P(x, y, z) & =C N^{-1 / 2} \exp \left(\frac{-x^{2}}{2\left\langle x^{2}\right\rangle}\right) N^{-1 / 2} \exp \left(\frac{-y^{2}}{2\left\langle y^{2}\right\rangle}\right) N^{-1 / 2} \exp \left(\frac{-z^{2}}{2\left\langle z^{2}\right\rangle}\right) \\
& \sim N^{-3 / 2} \exp \left(\frac{-3 r^{2}}{2\left\langle N a^{2}\right\rangle}\right),
\end{aligned}
$$

where $\mathbf{r}=\mathbf{a}_{1}+\mathbf{a}_{2}+\ldots+\mathbf{a}_{N}, \mathbf{r}^{2}=|\mathbf{r}|^{2}\left(|\mathrm{r}|^{2}=x^{2}+y^{2}+z^{2}\right)$ and $N$ is the step number.

We notice the following basic properties:
(1) The state of simple random walks describes a certain kind of uniform distribution of elements. This distribution can describe well the distribution of the seeds at the beginning of the evolution. This can be also supported by the fact that the Brownian motion can describe the simple random walks.
(2) We notice that Lévy theorem ensures the existence of symmetry breaking without any bias.

## Lévy-theorem ([7])

We consider a coin toss game. Then we have a 1-dim. random walk. We can have the following surprising fact: When we begin to win (i.e., we have an one-side appearance tendency), then the situation continues easily. Namely, when we win, we continue to win!! (see also ([29])).

## Application of Lévy-theorem to symmetry breaking ([29])

We can observe many symmetry breakings in nature: (1) The symmetry breaking between particles and anti-particles, (2) The uniformity breaking in the distribution of galaxies, (3) The difference of number of protons and neutrons in the atom generation, (4) The difference of numbers in egg-divisions and body-constructions. Usually we can not find the theoretical back ground of the symmetry breaking. Hence we say "because of some certain but unknown reasons, we have a symmetry breaking". When we accept Levi theorem, we can appreciate the symmetry breaking easily. The reason why we will not accept the symmetry breaking might be found in the traditional philosophy, which is called "The principle reason of sufficiency" (Leibniz). Also we have a strong belief in probability theory ([18], [26], [28]).

## 4. Self avoiding random walks in evolutions (Self organization: The power law distribution)

de Gennes has developed the theory of polymers in terms of the scale (critical exponent)" of the distribution of polymers ([8]). Also he put his mathematical foundation of his theory on self avoiding (s.a.) random walks. At first we recall some basic facts on s.a. random walk. Then we will see how s. a. random walk can describe polymer physics. We notice that the transition from simple random walk to self avoiding random walk decreases the entropies. We take a self-avoiding random walk on the lattice:


Self avoiding Random walk

Figure 14

When the path is constrained under the condition: The final point is the nearest neighbor point of the initial point (see, Figure 14). Then the self avoiding random walks give the following distribution with power law:

$$
p_{N}(a)=\left(1 / R^{d}\right) N^{(1-r)} .
$$

Here we notice that r is universal constant: $r \approx 7 / 6$ for $\operatorname{dim}=3, r \approx 4 / 3$ for $\operatorname{dim}=2$.

Remark. Simple random walks have been discussed widely. But we can find very few references on self-avoiding random walks.

## The phenomena of power law distribution and the evolutionary system

In the remainder of this section we will show how we can derive the power law distribution from the evolutionary system.
(1) Recently people have found many phenomena whose distribution make power law type.
Example 1: Zipf's law, Example 2: The distribution of galaxies, Example 3: Pareto's law, Example 4: The citation numbers of papers, Example 5: The distribution of populations of cities, Example 6: The distribution of the size of earthquakes.

Although we have so many examples and treatments of these phenomena we have still not its origin of the distributions ([35]).
(2) Next we will show how the evolutionary systems generate self-avoiding random walks and hence we may expect to treat the phenomena by the theory of evolutions. At first we notice that we can find phenomena which may be described in terms of self-avoiding random walks:

## Example 1: (Polymer generation)



## Example 2: (Transcription mechanism)



Example 3: (Cells of bacteria): We can find many examples of the body constructions of linear type:


Example 4 (Cosmology ([27])) We can expect to find the constructions of galaxies of the linear type. We can observe the type in the computer simulations:


Figure 15
Remark. This observation suggests a possibility of string theory in cosmology.
Example 5 (The power law distributions of sentences) Here we give the generation of sentences and we show that the distribution of sentences supply that of power law type. Then we may derive the well known Zipfs law on the base of this observation.

## Generation of sentences

We can find a linear structure in the generation of sentences by words:

$$
\text { I sleep } \Leftrightarrow \mathrm{I}+\text { "sleep, } \quad \text { I give you a book } \Leftrightarrow \mathrm{I}+\text { give }+ \text { you }+ \text { a book }
$$

Next we will show that the generation of sentences can be described in terms of the theory of the self avoiding random walk. We take a set of some units:

$$
Y_{1}, Y_{2}, Y_{3}, \ldots, Y_{n}
$$

which make a simple random walk. The evolution begins and they make a structure of the linear structure:

$$
Y_{1}+Y_{2} \Rightarrow Y_{1}+Y_{2}+Y_{3} \Rightarrow Y_{1}+Y_{2}+Y_{3}+Y_{4} \Rightarrow \ldots
$$

We notice that these sequences can be obtained by the successive binary extensions. Then we see that the set of these sequences generate self avoiding random walks. We can observe the BTBB-structure in the sequence of elements. We can describe it in terms of the block representation of binary extension, or ternary extension (see [16]):


Hence the BTBB structure can be observed in the following manner: For example

$$
\square \Rightarrow \begin{array}{|l|l|l|l|l|l|l|}
\hline & \\
& & & & & & \\
\hline
\end{array}
$$

(The words in the blocks are omitted). Hence we can make tapes of sequences in terms of block representations. This has been already given in Part III and the details are omitted here ([16]). By these observations, we can obtain sequences which constitute self avoiding random walks. Hence the distribution becomes the type of power law.

## Example 6 (The power law distributions in economy - Paretos law)

Here we will understand the Pareto's law from the evolutionary point of view. We consider the economy in the following steps: (1) The automaton structure of primitive gains of money, (2) The binary and ternary extensions in economics (the classic economy and the Keynesian economics) (3) The evolution in economics and the Pareto's law:

## (1) The automaton structure of primitive gains of money

We consider how people can gain/spend moneys. We may assume that moneys are floating and they constitute a simple random walk:


The primitive collection of moneys can be understood as making sequences of coins. In the most primitive level moneys are collected not for the practice use / commerce use, but for the presentation of social status. This can be observed in the shape of coins, namely, we have the holes on the center of coins. Hence we see that they have the structure of automatons (or regular languages):


Hence we can find the structure of self-avoiding random walk. Hence they make a distribution with power law.

## (2) The binary structure of collections of money

The next step might be the introduction of commerce. Then gain and expense of moneys both appear. We call gain of moneys as positive moneys, expense of moneys as negative moneys, respectively. We denote the scheme as follows:

$Y$ : positive money
$\underline{Z}$ : negative money
We may understand the pair as the binary extension of the collection of money in (1). The typical examples of negative moneys are usual payments for buying daily things or houses. Hence a perfect budget is the equality of the positive and negative moneys. Then people would be happy, when they have enough food daily to eat. This is an analogy of the acceptability condition of the basic binary sentence: $\mathrm{S}+\mathrm{V}$.

## (3) The ternary structure of collecting moneys

The most important character of gaining moneys is that positive monesy $Y$ and negative moneys $\underline{Z}$ create new moneys which are called benefit $W$ (or $\underline{W}$ ) when they are positive (resp. negative). This process can be understood as the ternary extension process. We can give examples of this process:
(1) $Y$ : capital, $Z$ : investment to innovation $\Rightarrow W$ : positive or negative benefit.
(2) $Y$ : money, $Z$ : investment to stocks $\Rightarrow W$ : benefit / loss

It is more convenient to understand this process by comparing the sentence generation.


Then following the analogy to the creation of sentences we can create moneys in a more complex manner. We can also observe the BTBB-structure in these processes. We may check this to be realistic or not. We may understand that this process can be understood as the creation of the self avoiding random walk and hence we may expect that the distribution becomes that of power law. We may expect to understand the Paretos law from these observations.

Remark: We may find binary and ternary structure in economy and construct the evolution theory for economy:

## The binary and ternary structures in economics

J. M. Keynes has discussed economics in terms of GDP. We can find the following binary and ternary structures:
(1) The binary structure in GDP

(2) Ternary structure in GDP


## 5. The fractal structure of the evolutionary system

Here we recall some basic facts on fractals and discuss the fractal structure of the evolutionary system in the following process:
(1) We will obtain the power law distribution for the evolutionary system.
(2) On the base of this fact we can consider the Tsallis entropy for the obtained exponent q of the system and make its q-analysis. Then we can find the roles of Tsallis entropy in the complexity systems ([35]).
(3) Finally we will be concerned with the phase transition of evolutionary systems. The system evolutes from the seeds of the evolution and creates the BTBB structure. Then the complexity system is generated and it describes the structure of "fractals of branched type". The boundary of the fractal set becomes fractal of flower type and it makes the state of the phase transition. By this we can describe several phase transitions in physics, cosmology and biology. We know that the boundary of the universe constitutes with black holes. Hence we see that the black holes may be understood as the flower of our universe". We can also describe the wall of cells as the flower of the generation of inner bodies (see, Figure 19). By these observations we can obtain the following description of evolutions:

## Fractal description of the evolution

(1) The seeds of the evolution are given
$\Downarrow$
(2) The complexity system is created from the BTBB-structure and successive binary extensions on the BTBB-structure. It becomes a fractal set of branch type
$\Downarrow$
(3) The border of the complexity system becomes a fractal set of flower type and it makes a simple random walk on the border (We may use the Tsallis entropy here)
(4) A new evolution begins from the seeds on the border.
$\Downarrow$

Next we will give the exact description of the fractal structure of the generating system.

## Some basic facts on fractal sets

We treat the following two kinds of fractal sets: (1) flower type (2) branch type:
(1) Fractal set of flower type: We choose a compact set $K_{0}$, for example, a closed interval and a system of contractions $\left\{s_{j}: K_{0} \rightarrow K_{0}: i=1,2, \ldots, M\right\}$. Then we put $\left\{K_{n}: n=1,2, \ldots, M\right\}$ by

$$
K_{n}=\bigcup_{j=1}^{m} s_{j}\left(K_{n-1}\right)
$$

Here we assume the separation condition $s_{i}\left(K_{n-1}^{0}\right) \cap s_{j}\left(K_{n-1}^{0}\right)=\emptyset(i \neq j)$, where $E^{0}$ is the open kernel of $E$. Then we have the sequence:

$$
K_{0} \supset K_{1} \supset K_{2} \supset K_{n} \supset \cdots .
$$

Putting $K=\bigcap_{n=0}^{\infty} K_{n}$, we have the fractal set of flower type. We notice that $K$ is $s_{j}$-invariant, i.e., $s_{j}(K)=K(j=1,2, \ldots, M)$.


Figure 16
(2) The fractal set of branch type (or evolution type): Let $K_{0}$ be a compact set and let $s_{j}(j=1,2, \ldots M)$ be a system of contractions on $K_{0}$ with the separation condition. Also we prepare a shift $s_{0}$ with $s_{0}\left(K^{0}\right) \cap K^{0}=\emptyset$. Putting

$$
K_{n}=K_{0} \cup s_{0}\left(\bigcup_{j=1}^{M} s_{j}\left(K_{n-1}\right), \quad n=1,2, \ldots\right.
$$

we have

$$
K_{0} \subset K_{1} \subset K_{2} \subset K_{n} \subset \cdots .
$$

Then we have the following relation between these fractals: We call the set: $K \backslash$ $\bigcup_{j=0}^{\infty} K_{j}$ the main boundary of $K$ which is denoted by $b K$. Then we can obtain the fractal set $b K$ of flower type. For the understanding the relationship between these two kinds of fractal sets, we give well known fractals of Cauliflower type. We may say that the boundary of the Cauliflower is just the flower of the Cauliflower.


Figure 17

## The fractal structure of the evolutionary systems

Applying the above observation to the evolutionary system, we can find the fractal structure of branch type on the evolutionary system and the fractal structure of flower type on its boundary respectively. We will describe the fractal structure of the evolutionary system in the following manner: We begin with the BTBB-system of the evolution which is denoted by $A_{0}$. Then the complexity system is generated by binary extension by $A_{i}(i=1,2, \ldots)$. Then we may find several types of the generations: For example,
(1) Linear type

(2) Planar type


## (3) Complexity type

We can find much more complicated fractal sets, for example, 3rd and 4th cells in molecular biology ([9]).


Figure 18
The fractal set of Hilbert type: We see that the body $A$ is dense in $A^{c l}$, where $A^{c l}$ is the closure of $A$, by definition. The fractal sets which have the condition: the boundary $\mathrm{bA}^{c l}$ is also dense in $\mathrm{A}^{c l}$ is called a fractal set of Hilbert type. We can discuss how "the parts and the whole" relate in organs in terms of the fractal set:
(1) The parts are equivalent to the whole. The sponge has the uniform structure. Hence, when it is divided, each part becomes a sponge (Example 1 in Figure 19).
(2) The parts are connected to the headquarter: the neuronal or blood system has a headquarter "brain/heart" and "nerve ending/capillary" (see Example 3 in Figure 19).

We consider the fractal set of the bounded type, i.e., the total fractal set is the bounded (compact) set, because the universe itself is bounded: $\bigcup_{j=0}^{\infty} A_{j}$ is a compact set. Hence we see that the diameter $\left|A_{n}\right| \rightarrow 0(n \rightarrow \infty)$.

## The power law distribution of the fractal sets

At first we notice that paths starting from the origin *make a self avoiding random walk. Hence we see that the distribution of paths makes that with power law. Also we notice that the fractal set of the boundary makes a simple random walk. Next we introduce the Hausdorff measures on A and bA, respectively, whose Hausdorff dimension is denoted by q and q', respectively. Then we can consider the Tsallis q-entropy on A and the Tsallis q'-entropy on bA, respectively ([35]).

Example 1 (Cells): We have the following two hypotheses on the creation of cells


Collection construction
We give several examples of creations:


Subdivision construction
(1) The linear construction

(3) The body construction
volvox

## Example 2 (Geometric type)



prokaryotic cell

eukaryotic cell

## Example 3 (The recurrence system)

The recurrence system is a typical system of Hilbert type.


Figure 19

Example 3 (Tool kit system) The tool kit is a method of the body construction which can be given by successive binary extensions. The genes which are called Hox genes operate on DNA as on-off mechanism and create the desired the body. Remarkable facts are that Hox genes have the universal sequence independent from the species and on-off mechanisms create the varieties of the life things. When the evolution proceed, then the duplications happen in Hox-genes and they can operate in a more complex manner. We may understand these evolutions by the binary succesive extensions:
(i) $\mathrm{A} \Rightarrow \mathrm{A}^{\prime}+\mathrm{A}(\mathrm{A}, \mathrm{A}$ is a sequence of DNA)
(ii) $\mathrm{A}+\mathrm{B} \Rightarrow(\mathrm{A}+\mathrm{B})$ (where B is a cell)
(iii) $\mathrm{A}+\mathrm{B} \Rightarrow(\mathrm{A}+\mathrm{B})$ or A ("switch on/off")

We give two examples:
(1) Fly
(2) Mouse


Figure 20

## 6. The evolutions of physics/cosmology

We will describe the BTBB-structures of the following three evolutions:
(1) The evolution of the space-time, (2) The evolution of the elementary particle physics, (3) The evolution of the generation of atoms and the universe.
(1) The evolution of the space-time:

## The BTBB-structure of the evolution of the space-time

(0) There exists the Penrose-Hawking singularity of the universe ([23], [27]). We may associate the origin of the evolution to the singularity. We may assume that the seeds of the space-time make a simple random walk (Process 1) (Remarks 1, $2,3)$.

$$
\Downarrow
$$

(1) The first binary extension creates our world and the ambient space by the Big- Bang and the Inflation. We call the ambient space "dark world (Remark 4). Here we want to notice that we have to assume the existence of the ambient space of this world (Remark 5).
$\Downarrow$
(2)The ternary extension creates the 3-dimensional space (Remark 6).
$\Downarrow$
(3) The second binary extension creates the time which separates the past and the future. Also it makes a simple random walk. The past and the future are still not determined (Remark 7).
$\Downarrow$
(4) The final binary extension introduces a concept of the entropy and the direction of the evolution is determined (Process II) (Remark 6).

Remark 1: The singularity is obtained by solving the Freedman equation ([27]):

$$
\frac{1}{a} \frac{d^{2} a}{d t^{2}}=-\frac{4 \pi G}{3 c^{2}}\left(\rho c^{2}+3 P\right)
$$

where $a$ : the scale factor of the universe, $\rho c^{2}$ : the mass density of the universe, $P$ : pressure.

Remark 2: The cosmological principle supports our assertion ([27]).
Remark 3: The quantization of the solutions of the Freedman equation gives the quantum gravity wave function $\Psi(a)$ which is given by the Wheeler-DeWitt equation:

$$
\left[-\hbar^{2} \frac{d^{2}}{d a^{2}}+\left(\frac{3 \pi}{2 G}\right)^{2}\left(a^{2}-\frac{a^{4}}{\ell^{2}}\right)\right] \Psi(a)=0
$$

where $\ell$ is horizon radius. Usually people treat the equation under the boundary condition of the separation of the space and time and obtain the real wave function by the tunnel effect. We suggest to set the boundary condition on the boundary between this world and dark ([23]).

Remark 4: The Lévy theorem says that there exist a symmetry breaking between this world and the dark world and nothing is determined and just fluctuated. After the entropy is introduced the flow of the evolution is determined.

Remark 5: It seems to authors peculiar that we do not assume the existence of the ambient world, because we cannot imagine an extension of this world without the ambient space. We may associate the dark matters and dark energy to the border of these worlds.

Remark 6: The quantized wave functions have no times. Hence they are stable waves. At this stage the 3 -dimensional space is created. This is the natural consequence of the potential function $f$ of the gravity which has the form $f \sim 1 / r$, where $r$ is the distance to the singularity. This is the basic observation on the gravity by Laplace ([27]). If we choose the other dimensions, then we see that the universe would be much poorer. The gravity has only the attractive power. This implies that the universe has begun with an explosion. This implies that each material has the common ancestor. This suggests us that we can introduce the concept of the gravity for other entropies. In fact, E. Verlinde has introduced a concept of entropic gravity and try to construct the relativity theory in the entropy level ([37]).


Figure 21
Remark 7: The Newtonian mechanics or Einstein theory cannot determine the past/ future. Some people try to deduce the past and future from the theory (for example, [23]). But they have not succeeded in it. We assert that the only entropy can describe the process from past to future.
(2) The evolution of elementary particles (From the birth to $10^{-33}$ sec.):
(1) At the Big Bang time the space is filled with photons. This is the origin of elementary particles. (2) The first binary extension creates particles and anti-particles: $\mathrm{q} \Leftrightarrow \mathrm{q}^{*}+\gamma$, where $\gamma$ is photon. They make simple random walk and they are fluctuated. By the Lévy theorem particles in this world appear for a period or antiparticles appear in the other period (Process I). (3) The ternary extension creates colors of quarks. Mesons and baryons are also created. (4), (5) The successive binary extensions create quark families: t-quark and b-quark are created at the third binary extension and c-quarks, s-qarks from t-quarks and u-quarks and d-quarks from b-quarks, respectively. We notice that weak bosons and leptons are created at the same time as the emission of entropy for the creation of baryons. This creation gives the duality between baryons and leptons.

Remark 8: The duality may be described in terms of the Galois group of the BTBB-structure. Also we may expect that the gauge theory cna be described in the analogous manner.
(3) The evolution of generations of atoms (From $10^{-33}$ sec. to 3 minutes):

After the creation of the quark family, the u-quark and d-quark remain. Then proton: p and neutron: n are created and they make an equilibrium state and make a simple random walk by $\mathrm{p} \Leftrightarrow \mathrm{n}+\mathrm{e}$, where e is an electron. (1) The origin of the evolution of atomic physics is the birth of protons and neutrons. They make a simple random walk and they make a symmetry breaking. (2) The first binary extension creates H and He by Gamov process (Process I). (3) The ternary extension creates C by Salpeter process. (4) The successive binary extensions create O, N, etc., by $\alpha$ process or $\beta$ decay process. At this stage the complexity system begins to be created and the light atoms are created until Fe (Process II) ([27]).

## (4) The birth of the stars and galaxies ( $3.8 \times 10^{5}$ years)

Before $3.8 \times 10^{5}$ years of the birth of the universe, electrons behave free from atoms because of the high temperature. The photon states and material states make an equilibrium state and atoms behave as free particles. They make a simple random walk which has been observed as black radiation by COBE. This can be chosen as the first stage of the evolutions of the universe ([27]).


Figure 22

From this stage, the same process in the previous process (3) creates stars under the effect of gravity powers (see Remark). For example, light stars in the principal sequence are created by Gamov process (Process I). This process continues to the creation of rather heavier stars which are created by Salpeter process and the successive binary extensions. The quite heavy stars, hypernova are created and they grow to black holes (see Figure 24). Then the stars are floating in the universe and they make a simple random walk (Process I'). Next the Gamov process and the Salpeter process happen repeatedly and the galaxies are created. Then the complexity system is created and we can observe the non-homogenous distribution of the galaxies (see Figure 23).


Figure 23
At present, the border between our universe and the ambient universe is filled with black holes and dark energy. The dark energy might be the emission of energy of the creation of stars and galaxies. Then stars and galaxies will be absorbed into the border of the universe and the final Big Crunch will happen and the end of this universe creates the birth of the new universe (Process II).

## Example (Black holes)



Figure 24

Remark. We can find a new ternary extension process. The generations of stars and galaxies are performed by the same process of the atom generations. At the end of Section 8 we make a comparison between physics and biology.

## 7. The evolution in polymer physics (de Gennes theory on polymers)

de Gennes developed the theory on polymers. His main idea is the scaling assumption on the distribution of polymers. The heart of his theory is the application of the self avoiding random walk to polymers. We give the basic generation table of polymers connected hydrocarbon polymer. Then we can find binary and ternary structures in polymers and find its evolution theory.

## (1) The binary generation of polymers


(2) The ternary generation of polymers


Figure 25

## The complexity system of polymers

At first we recall the basic idea on de Gennes theory on polymers. We can associate polymers to random walks in the following manner:
(I) We consider a large box filled with balls of the same size without free space.


Figure 26
(i) We choose a referred ball O and move to the right. Then the empty space of the balls is filled with some another ball. Hence we can move to the right.
(ii) Repeating this process the ball can make a simple random walk on the lattice.
(II) Next we consider a polymer which constitutes with several (long) monomers with two hands for simplicity sake

$$
-\mathrm{O}-\mathrm{O}-\mathrm{O}-\mathrm{O}-\mathrm{O}-\mathrm{O}-
$$

Here we assume that the polymer is easily bended. Then it makes a similar move to that of monomers. But it can not make self-interactions. Hence it makes a selfavoiding random walk. On the basis of these facts, de Gennes has developed the polymer physics. The key point of his idea is the appearance of the distribution of power law. By this he developed the scaling theory of polymers. Here we consider the complexity system of polymers and try to find the binary and ternary structures in polymers.
(1) At first we notice that these polymers make self avoiding random walks and that their distributions constitute those of power law.
(2) Polymers constitute knots/braid structures


Figure 27
Here we notice that we may expect to describe the structure in terms of knot structure. In fact, we know the binary and ternary structures in the generation of knots/braides can be given by Reidemeister moves. Comparing the structures of linear polymers with those of knots/braids, we may find binary and ternary structures in the complex structure of the polymers. This will be discussed in Part V ([17]).

Reidemeister move (I), (II), (III) $\Leftrightarrow \alpha$-spiral structure, $\beta$-spiral structure, collagen 3 -line spiral, respectively (see Figure 27).
(3) The entropy of the states of polymers plays an essential role for the description of the states (see Figure 28).


Figure 28

## The evolution theory of polymers

We can treat the evolution theory of polymers in our scheme (see Figure 29).


Figure 29

## 8. The evolutions in biology

In this section, we will treat evolutions in biology and find binary and ternary structures in them and their generations of complex systems. Then we can find the essential change in the evolution from molecular biology level to system biology level. Comparing the both evolutions in physics/cosmology and biology, we can find the possibility of system physics/cosmology. We begin with the evolutionary tree of biology.


Figure 30
Following our method we can find three stages of evolutions (see Figure 30):
(I): The chemical evolution
(II-1): The biological evolution (The birth of life)
(II-2): The biological evolution (Generation of big-size cells)
(III): The biological evolution (The birth of system biology)

## (I) The chemical evolution

This evolution is a continuation of that of polymers. About $35 \times 10^{8}$ years before, the basic material in biology i.e. RNA, DNA and proteins are created. We can observe the binary and ternary extension structures in these generations. We can observe the BTBB-structure in the generation of RNA, DNA and proteins.

## The BTBB-structure and complexity system in the chemical evolution

(1) The origin of the evolution is the RNA
$\Downarrow$
(2) The first binary extension creates DNA $\Downarrow$
(3) The ternary extension creates proteins
$\Downarrow$
(4) The successive binary extensions create the variety of proteins
$\Downarrow$
(5) The final binary extension creates the entropy which makes the difference between "Inner world" (body) and "Outer world" (environment) of the cells

Remark: The mutations in the cell make a neutral evolution ([18]). Hence we see that the set of proteins make a simple random walk.

## (II-1) The biological evolution (The birth of life)

The origin of the birth of lives exists but it is unclear ([3]).

## The BTBB-structure and complexity system in the biological evolution (II-1)

(1) The origin of the evolution is the creation of life $\Downarrow$
(2) The first binary extension creates the duplication of genes (EF-1 and EF-2) $\Downarrow$
(3) The ternary extension creates 3 -domains (archeas, bacteria eukaryota) $\Downarrow$
(4) The succesive binary extensions create the organization of cells $\Downarrow$
(5) The effect of the entropy creates cells, but the borders are not necessarily clear

Remarks: (1) The evolution of this state is supported by genome analysis by Woose and Iwabe, etc ([11]). (2) The horizontal evolutions happen frequently between cells.

## (II-2) The biological evolution (The birth of big and complicated cells)

Choosing the same process in (II-1) with additional various binary extensions, we see that the size of the cells have become bigger and more complicated in this evolution.

## The BTBB-structure and complexity system in the biological evolution (II-2)

We can observe the BTBB-structure in the birth of big cells.
(1) The beginning of the evolution is the final state of (II-1)
$\Downarrow$
(2) Then the first binary extensions happen (symbiotic evolutions, tool-kit evolutions duplications, mutations, etc...).
$\Downarrow$
(3) The first ternary extension creates 3-families (parazoa type, deuterosome type, protosome type).
$\Downarrow$
(4), (5) Then the successive binary extensions create the organizations of cells.

Remarks. (1) We can find the same kinds of other evolutions which create the big cells, replacing (3) with the following ternary extension: (3)' The creation of 3 -families (animal, plant, fungi).
(2) We can observe many kinds of binary extensions which generate many structures, for example, (1) The appearance of sex, (2) The generation of duplication ([21]). (3) The appearance of the tool kit mechanism (as for the mechanism, see Section 5). (4) The symbiotic evolution (L. Margulis) ([9]). Hence it is natural to think that some of them are chosen for the completion of the BTBB-structure. After succesive binary extensions, we have obtained cells with complex structures (see Section 5). We want to notice that these evolutions can be decsribed in terms of succesive binary extensions.
(3) Whitteker 5 world theory or Margulis 5 world theory describes the evolution of this stage ([9]).

## (III) The biological evolution (The birth system of biology)

At the end of the second stage multi cells are constructed. Moreover, the structure of the cells is enough complicated. The luxury genes appeared and multi-cells make groups and began to create the total body (see Section 5, Figure 19). We notice
that there happens essential change in the evolution. The evolutions performed not in the genes level, but in the system of genes. So we have to treat them in terms of system biology. Then we have the "Big Cambrian Explosion. We will find the BTBB-structure in this level.

## The BTBB-structure and complexity system in the biological evolution (III)

(0) The life things with big sized cells have lived without enemies and they enjoyed their lives very much. This implies that they made a simple random walk in the "space of shapes or forms" (Huxley's apple barrel).
$\Downarrow$
(1) The first binary extension: We can see that luxury genes are created and they create the system of cells.
$\Downarrow$
(2) The first ternary extension: We may choose the following three types in the body form level as in the previous stage: (i) The parazoa type (radial symmetry) (ii) The deuterstome type (linear symmetry) (iii) The protostome type (Up mod down, symmetry)
$\Downarrow$
$(3) \sim(5)$ The successive binary extensions. The influences from the outer/inner world changes the body from "non-understandable" forms" to reasonable".

Remark 1: The set of multi cells makes a simple random walk. Typical examples are bolbox, or nostoc (see Figure 19, Section 5).

Remark 2: We give several examples of each type.

(1) The parazoa type

(2) The deuterostome type

(3) The protosomo type

In the Cambrian years, the animals with complicated and non-understandable structures appeared. We may see that these full non-deterministic structures make a simple random walk in the space of varieties of forms. We give several examples of changes from non-understandable forms to understandable forms.

（1）non－understandable forms
$\Downarrow$

vetulicola

pikaia

C）
mouse gill
（2）understandable forms

Figure 32

## Summary and future problems （A method of system analysis to physics／cosmology）

Here we will summarize the results of this paper and shall try to find a method of system analysis to physics／cosmology．We have discussed the evolutions by non－ commutative Galois theory and we have found the following two stages：
（I）The generation of BTBB－system and the complexity system
（II）The repetition of the process（I）creates the total hierarchy structure of the evolutionary system，performing this process recursively and we have obtained the hierarchy system $\mathscr{L}_{n}(n=1,2 \ldots)$ ，（see Remark in the Introduction）．

The process（I）can be detected by finding the ternary extension．Then we can find the process（II）．We can observe the fact that the evolution of（I）is drastic and new．The evolution becomes systematized in（II）on the base of the results of the evolution in（I）．Hence we can find the curriculum vitae of the evolution．The evolution（I）（resp．（II））is called young（resp．adult）evolution．We have observed both young and adult evolutions in an evolution．

Our main interest is to know at what stage the referenced stage places in the evolution process．Namely，we want to know wheather the referenced stage is young， adult or old．

## System biology vs system physics

As we have seen in Section 8, the observation in biology tells that there exists a big change from the molecular biology (the stage (I), (II)) to system biology (the stage (III)). The same kind of the change can be also observed in the evolution of the universe. In fact, we cannot find a new ternary extension in the evolution of stars and galaxies (see Remark below Figure 24). Hence we may understand thus the creations of stars and galaxies perform along the same creation process of atoms. We may understand that the atomic level evolution completed and the evolution of the total system of stars and galaxies began under the influence of gravity. Hence we may have the following comparison table:

| Biology | Physics/Cosmology |
| :--- | :--- |
| (1) Molecular generation (RNA, DNA) | (1') Elementary particles generation |
| (2) Cell creation | (2') Atom creation |
| (3) Big and complicated body | (3') Stars and galaxies |
| (Luxury genes) | (Gravity effect) |

Problem: Is the present universe young, adult or old? Will/did there exist the Cambrian explosion in the universe?

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## STRUKTURY BINARNE I TERNARNE W EWOLUCJI WSZECHŚWIATA (ŚWIAT $2 \times 3 \times 2 \times \ldots$ WYMIAROWY) IV ENTROPICZNY OPIS EWOLUCJI

## Streszczenie

Niniejszy artykuł jest czwartą czȩścią artykułów napisanych pod tym samym tytułem [30, 31, 33]. W pierwszej i w drugiej czȩści widzieliśmy, że struktury binarne i ternarne mogą opisywać ewolucjȩ systemów, na przykład kwarków, atomów, galaktyk, RNA, DNA i jȩzyków. W trzecim artykule przedstawiliśmy ewolucjȩ jȩzyków i pokazaliśmy, że ma ona ścisły związek z ta̧ w fizyce. W tej czȩści rozwiniemy najpierw ogólną teorię ewolucji dla systemów o strukturach binarnych i ternarnych. Nastȩpnie pokażemy, jak systemy ewolucyjne tworzą tak zwane systemy złożoności jako granicȩ systemu ewolucyjnego. Rozważamy ewolucjȩ w oparciu o nastȩpujạce zasady:

## Zasady ewolucji:

(1) Wszystko w tym wszechświecie musi podlegać prawu wzrostu entropii (zasadzie Boltzmanna) ([35]);
(2) Systemy ewolucyjne działajạ wbrew zasadzie Boltzmanna (zasadzie Schrödingera lub filozofii Bergsona) ([3]).

Stowa kluczowe: entropia, bła̧dzenie losowe, ewolucja wszechświata, nieprzemienna teoria Galois

